Concrete / Pictorial / Abstract Maths Calculation Policy

This policy has been largely adapted from the White Rose Maths Hub Calculation Policy with further material added. It is a working document and will be revised and amended as necessary. Many variations have been included to provide teachers with a range of tools to support pupils in their grasp of number and calculation. To ensure consistency for pupils, it is important that that the mathematical language used in maths lessons reflects the vocabulary used throughout this policy.

Recommended practice delivering a mastery approach

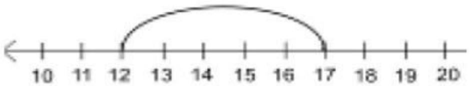
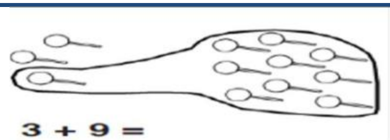
Warren Wood defines mastery as a deep understanding of mathematical concepts. True mastery aims to develop all children's mathematical understanding at the same pace.

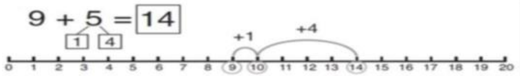
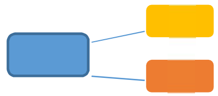
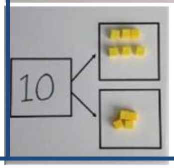
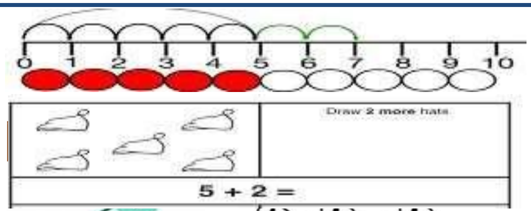
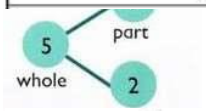
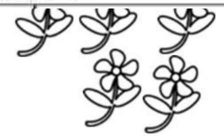
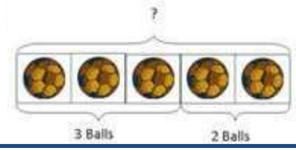
Consistency in language is essential for pupils to understand the concepts presented in mathematics. If other, 'child-friendly' terminology is used, this must be alongside the current terminology recommended by maths specialists.

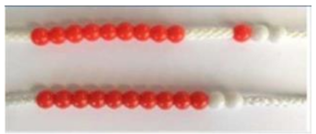
Concrete, pictorial, abstract (CPA) is an approach to be used with the whole class and teachers should promote each area as equally valid. Manipulatives in particular must not solely be presented as a resource to support the less confident or lower attaining pupils.

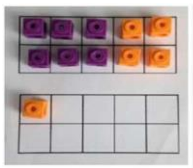
The abstract should run alongside the concrete and pictorial stage as this enables pupils to better understand mathematical statements and concepts.

YEAR 1 Addition

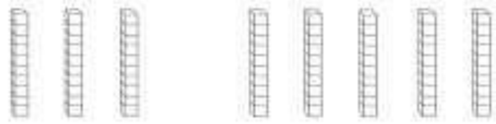
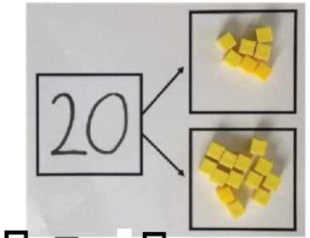
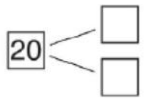
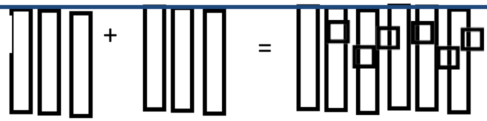
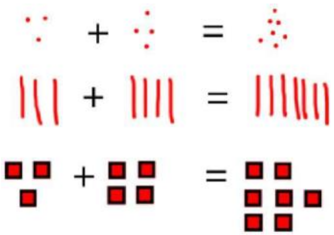
Objective / Strategy	Concrete	Pictorial	Abstract
Combining two parts to make a whole: part- whole model	  Use part, part whole model. Use cubes to add two numbers together as a group or in a bar.	Use pictures to add two numbers together as a group or in a bar. 8 1	8 = 5 + 3 5 5 + 3 = 8 3 Use the part part whole diagram as shown above to move into the abstract. Include missing number questions to support varied fluency: 8 = ? + 3 5 + ? = 8
Starting at the bigger number and counting on	 Start with the larger number on the bead string and then count on to the smaller number 1 by 1 to find the answer.	 12 + 5 = 17 Start at the larger number on the number line and count on in ones or in one jump to find the answer.	5 + 12 = 17 Place the larger number in your head and count on the smaller number to find your answer.
Regrouping to make 10. *This is an essential skill for column addition later.*	6 + 5 = 11 Start with the		7 + 4= 11 If I am at seven, how many more do I need to make 10? How many more do I add on now?

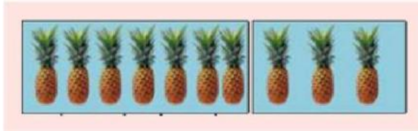
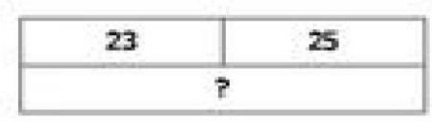
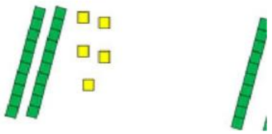
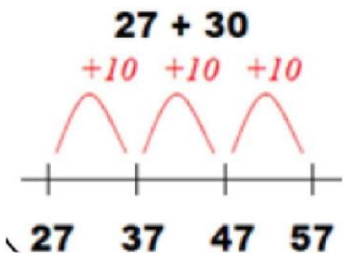
	bigger number and use the smaller number to make 10. Use ten frames.	Use pictures or a number line. Regroup or partition the smaller number using the part, part whole model to make 10.  	
Represent & use number bonds and related subtraction facts within 20	 2 more than 5.   	    	Include missing number questions: $8 = ? + 3$ $5 + ? = 8$ Emphasis should be on the language *'1 more than 5 is equal to 6.'* *'2 more than 5 is 7.'* *'8 is 3 more than 5.'*

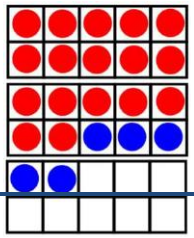
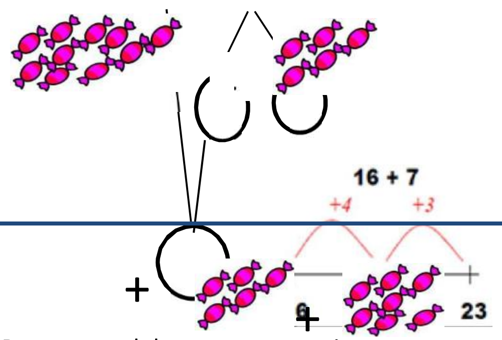
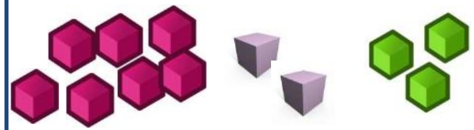
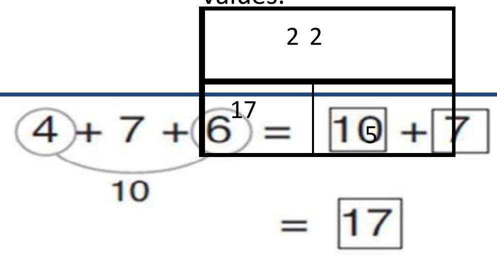


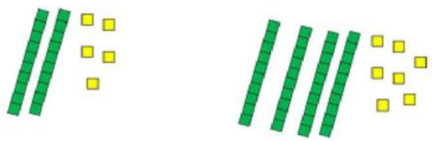


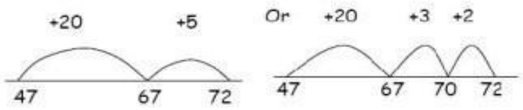
YEAR 2 Addition

Objective /Strategy	Concrete	Pictorial	Abstract
Adding multiples of ten	50= 30 = 20  Model using dienes and bead strings	 3 tens + 5 tens = ____ tens 30 + 50 = ____ Use representations for base ten.	20 + 30 = 50 70 = 50 + 20 40 + □ = 60
Use known number facts Part, part whole	 Children explore ways of making numbers within 20	 □ + □ = 20 20 − □ = □ □ + □ = 20 20 − □ = □	Explore commutativity of addition by swapping the addends to build a fact family. Explore the concept of the inverse relationship of addition and subtractions and use this to check calculations. □ + 1 = 16 16 − 1 = □ 1 + □ = 16 16 − □ = 1
Using known facts	 + =	 Children draw representations of H,T and O	3 + 4 = 7 *leads to* 30 + 40 = 70 *leads to* 300 + 400 = 700

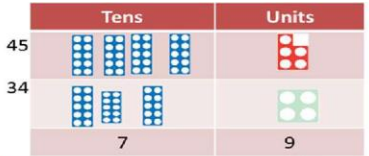
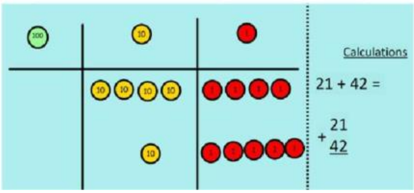
Bar model	 3 + 4 = 7	 7 + 3 = 10	 23 + 25 = 48
Add a two digit number and ones	17 + 5 = 22 Use ten frame to make 'magic ten Children explore the pattern. 17 + 5 = 22 27 + 5 = 32	17 + 5 = 22 Use part part whole and number line to model. 3 2 20	17 + 5 = 22 Explore related facts 17 + 5 = 22 5 + 17 = 22 22—17 = 5 22—5 = 17 Lead into recording in column format, to reinforce place value and prepare children for formal written methods with larger values.
Add a 2 digit number and tens	 25 + 10 = 35 Explore that the ones digit does not change	 	27 + 10 = 37 27 + 20 = 47 27 + □ = 57

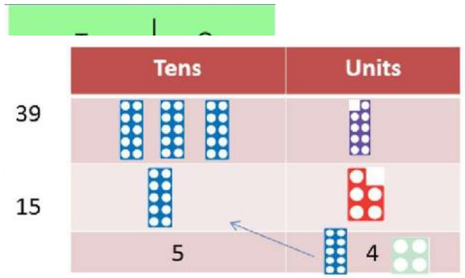
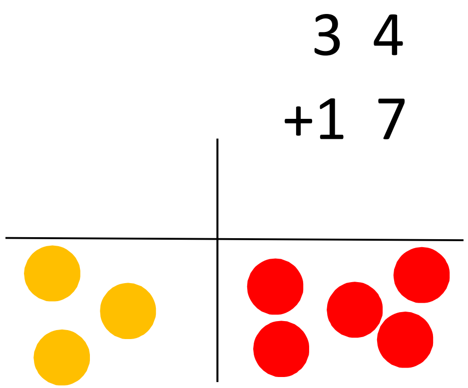
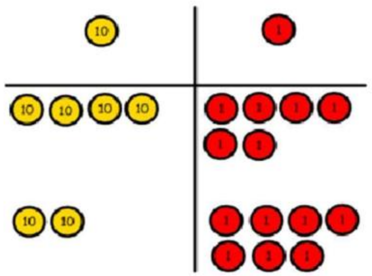
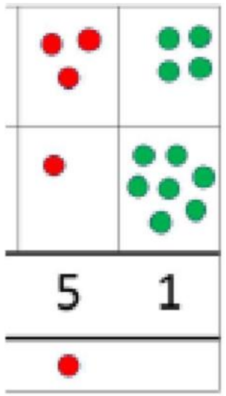
Add two 2-digit numbers	Model using dienes , place value counters and numicon	Use number line and bridge ten using part whole if necessary.	25 + 47 20 + 5 40 + 7 20 + 40 = 60 5+ 7 =12 60 + 12 = 72
			Lead into recording in column format, to reinforce place value and prepare children for formal written methods with larger values.
Add three 1-digit numbers	 Combine to make 10 first if possible, or bridge 1o then add third digit	Regroup and draw representation. + = 15	 Combine the two numbers that make/ bridge ten then add on the third.





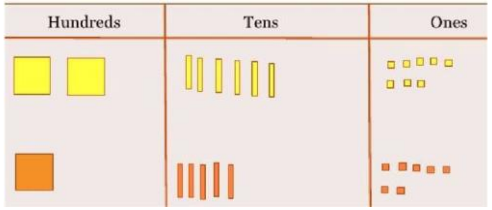
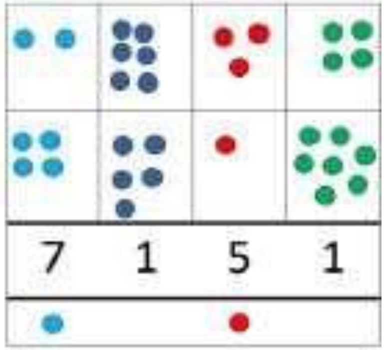
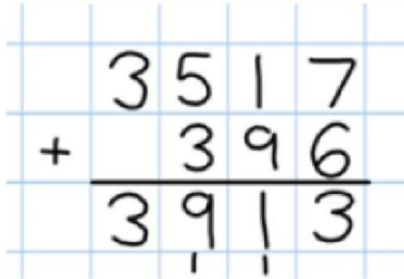
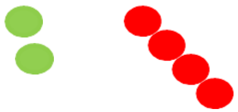
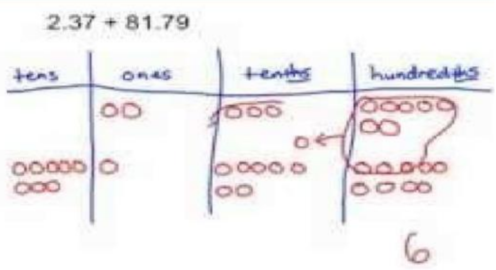
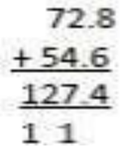
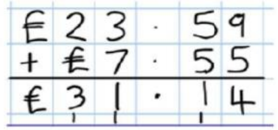
YEAR 3 Addition

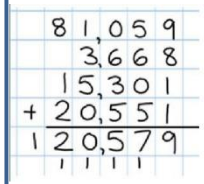
Objective /Strategy	Concrete	Pictorial	Abstract
Column Addition—no regrouping (friendly numbers) Add two or three 2 or 3digit numbers.	Dienes or numicon Add together the ones first, then the tens.   Move to using place value counters	Children move to drawing the counters using a tens and one frame. tens ones	$\begin{array}{r} 2\ 2\ 3 \\ +\ 1\ 1\ 4 \\ \hline 3\ 3\ 7 \end{array}$ Add the ones first, then the tens, then the hundreds.

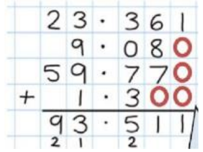
Column Addition with regrouping.	 Exchange ten ones for a ten. Model using numicon and place value counters.	 Children can draw a representation of the grid to further support their understanding, carrying the ten **underneath** the line	$20 + 5$ $40 + 8$ $60 + 13 = 73$ Start by partitioning the numbers before formal column to show the exchange.
	 46 + 27 = 73		$\begin{array}{r} 536 \\ +\ 85 \\ \hline 621 \\ \hline 1\,1 \end{array}$

Estimate the answers to questions and use inverse operations to check answers	 Estimating 98 + 17 = ? 100 + 20 = 120	Use number lines to illustrate estimation.  	Building up known facts and using them to illustrate the inverse and to check answers: 98 + 18 = 116 116 – 18 = 98 18 + 98 = 116 116 – 98 = 18

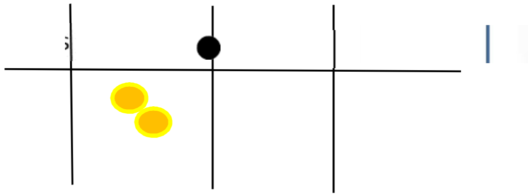
YEARS 4 – 6 Addition

Objective /Strategy	Concrete	Pictorial	Abstract
Years 4 – 6 Estimate and use inverse operations to check answers to a calculation	**AS per Year 3**		
Y4—add numbers with up to 4 digits	Children continue to use dienes or place value counters to add, exchanging ten ones for a ten and ten tens for a hundred and ten hundreds for a thousand.  	 Draw representations using place value grid.	 Continue from previous work to carry hundreds as well as tens. Relate to money and measures.
Y5—add numbers with more than 4 digits. Add decimals with 2 decimal places, including money.	As year 4 Ten ones tenths hundredths  Introduce decimal place value counters and model exchange for addition.	 	 

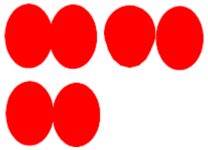
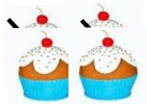
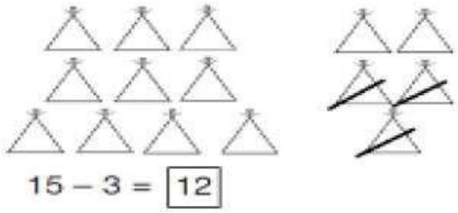
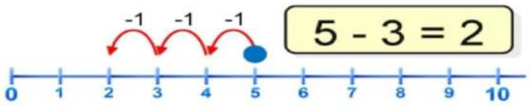
<table>
<tr>
<td>Y6—add several numbers of increasing complexity, including adding money, measure and decimals with different numbers of decimal points.</td>
<td>As Y5</td>
<td>As Y5</td>
<td>Insert zeros for place holders.


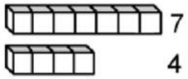
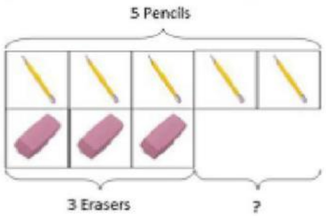
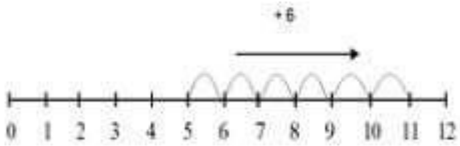


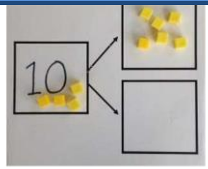
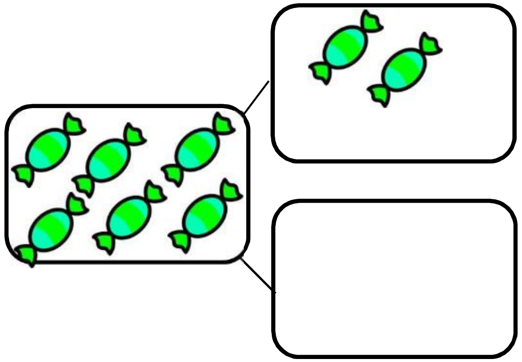
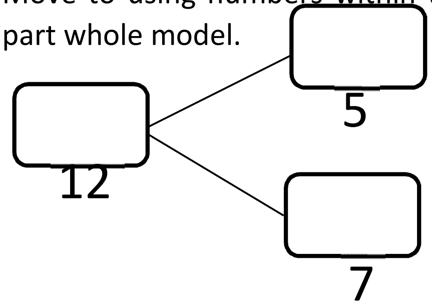
</td>
</tr>
</table>

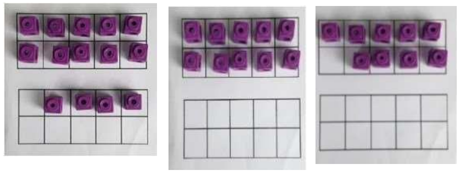
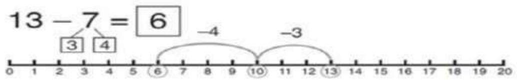
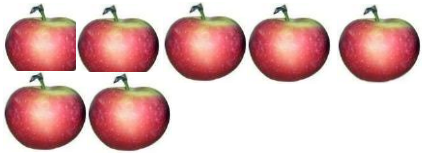


YEAR 1 SUBTRACTION

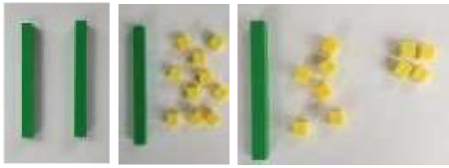
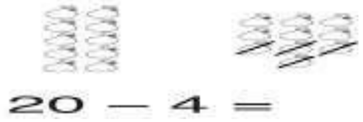
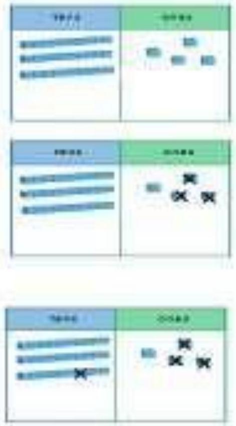
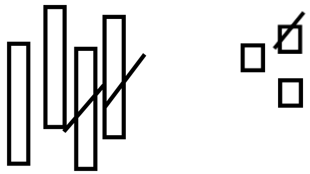
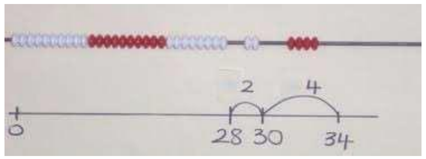
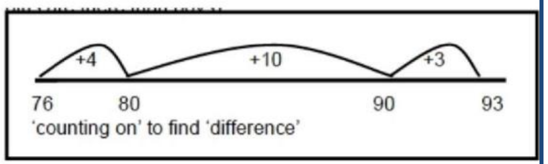
Objective /Strategy	Concrete	Pictorial	Abstract
Taking away ones.	Use physical objects, counters, cubes etc to show how objects can be taken away. 4—2 = 2   6—4 = 2	Cross out drawn objects to show what has been taken away.  	7—4 = 3 16—9 = 7
Counting back	  Move objects away from the group, counting backwards.  Move the beads along the bead string as you count backwards.	 Count back in ones using a number line.	Put 13 in your head, count back 4. What number are you at?

Find the Difference	Compare objects and amounts  'Seven is 3 more than four' 'I am 2 years older than my sister'  Lay objects to represent bar model.	Count on using a number line to find the difference. 	Hannah has12 sweets and her sister has 5. How many more does Hannah have than her sister.?

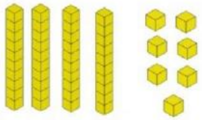
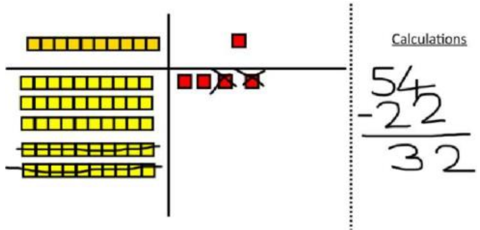
Objective/Strategy	Concrete	Pictorial	Abstract
Represent and use number bonds and related subtraction facts within 20 **Include subtracting zero** Part Part Whole model	 Link to addition. Use PPW model to model the inverse. If 10 is the whole and 6 is one of the arts, what s the other part? 10—6 = 4	 Use pictorial representations to show the part.	Move to using numbers within the part whole model.  Include missing number problems: 12 - ? = 5 7 = 12 - ?

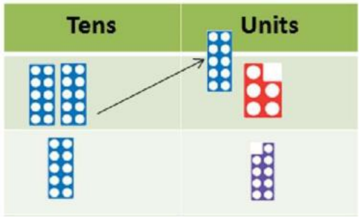
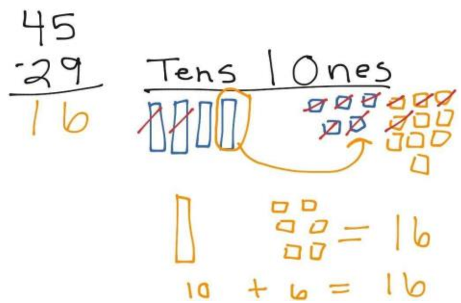
Make 10	14—9  Make 14 on the ten frame. Take 4 away to make ten, then take one more away so that you have taken 5.	 13—7 Jump back 3 first, then another 4. Use ten as the stopping point.	16—8 How many do we take off first to get to 10? How many left to take off?
Bar model Including the inverse operations.	 5—2 = 3		8 2 10 = 8 + 2 10 = 2 + 8 10—2 = 8 10—8 = 2

YEAR 2 - SUBTRACTION

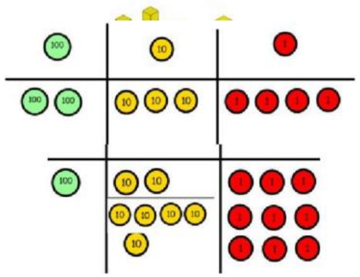
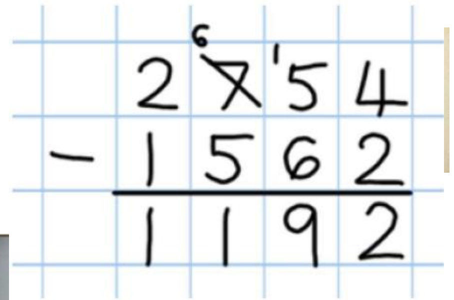
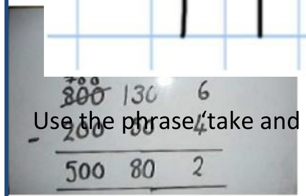
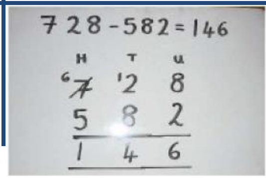
Objective & Strategy	Concrete	Pictorial	Abstract
Regroup a ten into ten ones	 Use a PV chart to show how to change a ten into ten ones, use the term 'take and make'		20—4 = 16
Partitioning to subtract without regrouping. *'Friendly numbers'*	34—13 = 21  Use Dienes to show how to partition the number when subtracting without regrouping.	Children draw representations of Dienes and cross off.  43—21 = 22	43—21 = 22
Make ten strategy *Progression should be crossing one ten, crossing more than one ten, crossing the hundreds.*	 34—28 Use a bead bar or bead strings to model counting to next ten and the rest.	 Use a number line to count on to next ten and then the rest.	93—76 = 17

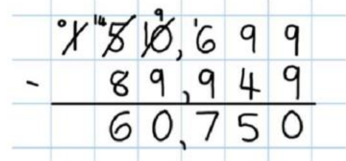
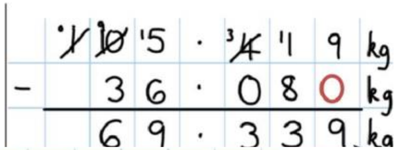
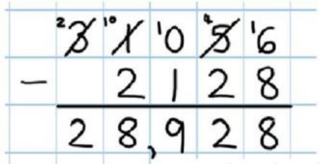
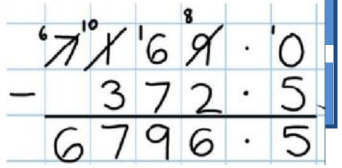
YEAR 3 - SUBTRACTION

Objective/ Strategy	Concrete	Pictorial	Abstract
Subtract numbers mentally, including: three digit number + ones three digit number + tens three digit number + hundreds			Vary the position of the answer and question. Expose children to missing number questions and vary the missing part of the calculation. 678 = ? - 1 688 – 10 = ? 678 = ? – 100
Column subtraction without regrouping (friendly numbers)	 47—32 Use base 10 or Numicon to model	 Draw representations to support understanding	47 – 24 = 23 40 + 7 – 20 + 4 20 + 3 Intermediate step may be needed to lead to clear subtraction understanding.

Column subtraction with regrouping	 Begin with base 10 or Numicon. Move to pv counters, modelling the exchange of a ten into ten ones. Use the phrase ‘take and make’ for exchange.	 Children may draw base ten or PV counters and cross off.	Begin by partitioning into pv columns Then move to formal method.

YEARS 4 – 6 SUBTRACTION

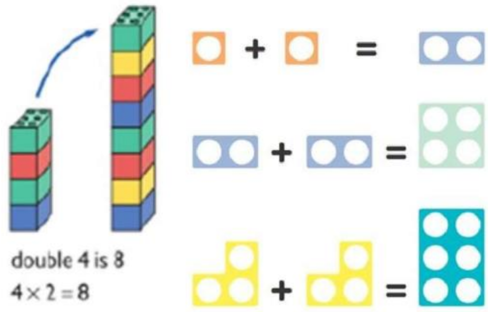
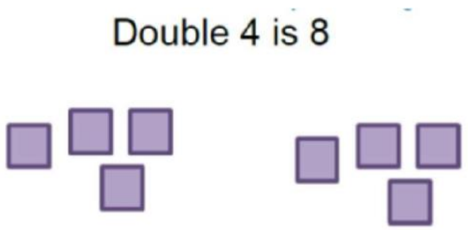
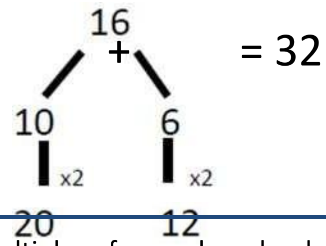
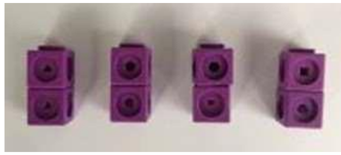
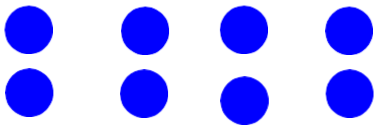
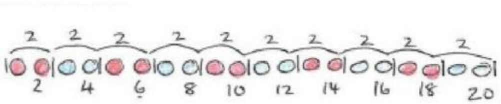
Objective /Strategy	Concrete	Pictorial	Abstract
Subtracting tens and ones Year 4 subtract with up to 4 digits. *Introduce decimal subtraction through context of money*	234 - 179  Model process of exchange using Numicon, base ten and then move to PV counters.	Children to draw pv counters and show their exchange—see Y3	 Use the phrase ‘take and make’ for exchange
Year 5- Subtract with at least 4 digits, including	As Year 4	Children to draw pv counters and show their exchange—see Y3	

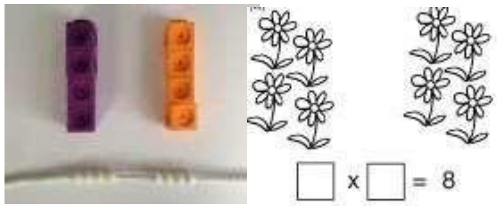
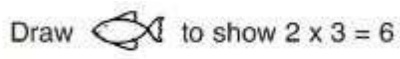
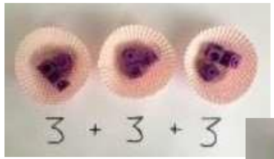
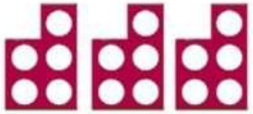
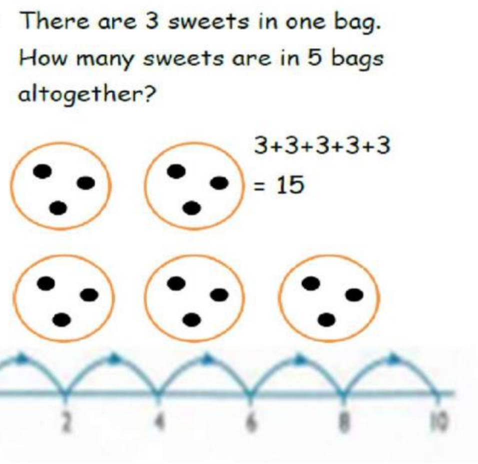
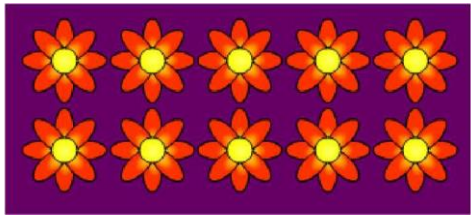
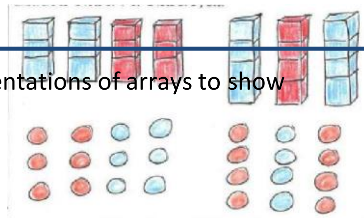
money and measures. *Subtract with decimal values, including mixtures of integers and decimals and aligning the decimal Up to 3 decimal places*			Use zeros for placeholder s.
Year 6—Subtract with increasingly large and more complex numbers and decimal values (up to 3 decimal place).	As Year 4	Children to draw pv counters and show their exchange—see Y3	   Use zeros for placeholder s. 

YEAR 1 MULTIPLICATION

Programme of Study specifies the following objectives, however it does not require the explicit teaching of the m

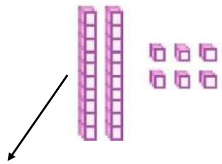
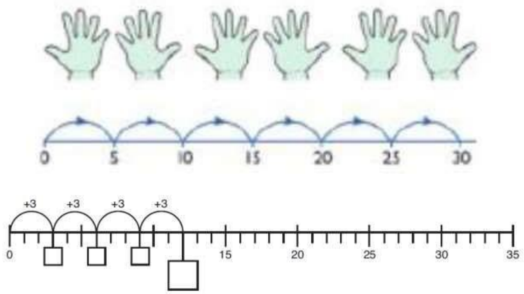
Objective / Strategy	Concrete	Pictorial

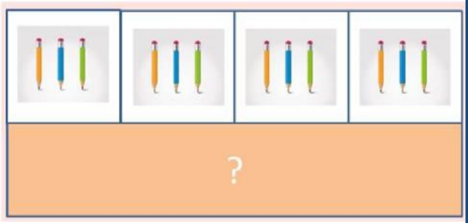
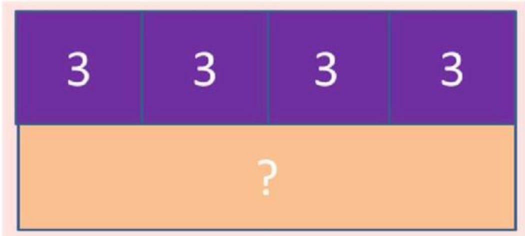
Doubling	Use practical activities using manipultives including cubes and Numicon to demonstrate doubling 	Draw pictures to show how to double numbers 	Partition a number and then double each part before recombining it back together. 
Counting in multiples (2s, 5s, 10s)	Count the groups as children are skip counting, children may use their fingers as they are skip counting.  	 Children make representations to show counting in multiples. 	Count in multiples of a number aloud. Write sequences with multiples of numbers. 2, 4, 6, 8, 10 5, 10, 15, 20, 25 , 30

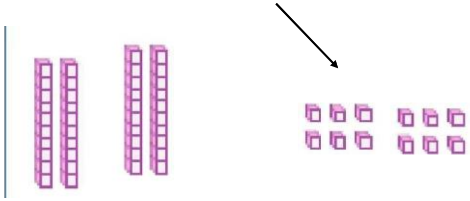
Making equal groups and counting the total	 Use manipulatives to create equal groups.	 Draw and make representations prob	2 x 4 = 8
Repeated addition	   Use different objects to add equal groups	 	Write addition sentences to describe objects and pictures. 
Understanding arrays	Use objects laid out in arrays to find the answers to 2 lots 5, 3 lots of 2 etc. 	Draw representations of arrays to show  understanding	3 x 2 = 6 2 x 5 = 10

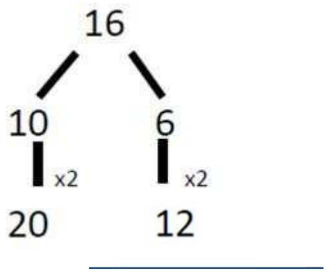
YEAR 2 MULTIPLICATION

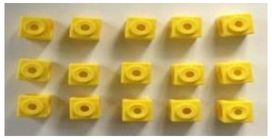
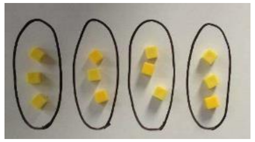
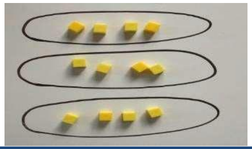
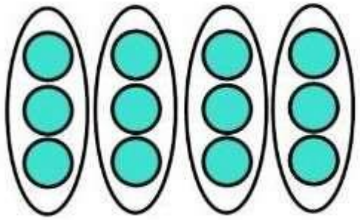
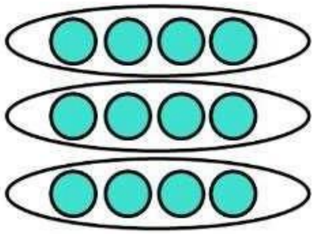
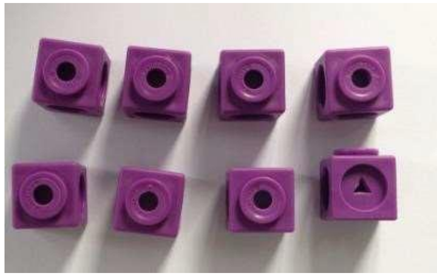
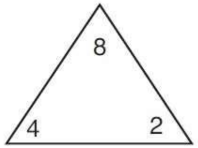
Children should be able to recall and use multiplication and division facts for the 2, 5 and 10 times times tables.

Objective / Strategy	Concrete	Pictorial	Abstract
Doubling	Model doubling using dienes and PV counters.  40 + 12 = 52	Draw pictures and representations to show how to double numbers	Partition a number and then double each part before recombining it back together. + = 32
Counting in multiples of 2, 3, 4, 5, 10 from 0 (repeated addition)	Count the groups as children are skip counting, children may use their fingers as they are skip counting. Use bar models.   5 + 5 + 5 + 5 + 5 + 5 + 5 + 5 = 40	Number lines, counting sticks and bar models should be used to show representation of counting in multiples. 	Count in multiples of a number aloud. Write sequences with multiples of numbers. 0, 2, 4, 6, 8, 10 0, 3, 6, 9, 12, 15 0, 5, 10, 15, 20, 25 , 30

			$4 \times 3 = \square$





Objective / Strategy	Concrete	Pictorial	Abstract
Multiplication is commutative	Create arrays using counters and cubes and Numicon.    Pupils should understand that an array can represent different equations and that, as multiplication is commutative, the order of the multiplication does not affect the answer.  	Use representations of arrays to show different calculations and explore commutativity.  	12 = 3 × 4 12 = 4 × 3 Use an array to write multiplication sentences and reinforce repeated addition.  5 + 5 + 5 = 15 3 + 3 + 3 + 3 + 3 = 15 5 x 3 = 15 3 x 5 = 15
Using the Inverse *This should be taught alongside division, so pupils learn how they work alongside each other.*		 $\square \times \square = \square$ $\square \times \square = \square$ $\square \div \square = \square$ $\square \div \square = \square$	2 x 4 = 8 4 x 2 = 8 8 ÷ 2 = 4 8 ÷ 4 = 2 8 = 2 x 4 8 = 4 x 2 2 = 8 ÷ 4 4 = 8÷ 2

			Show all 8 related fact family sentences.

YEAR 3 MULTIPLICATION

Children should be able to recall and use multiplication facts for the 3,4, and 8 times tables

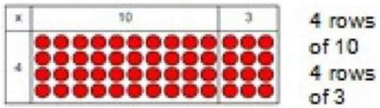
Objective /Strategy

Grid method, progressing to the formal method

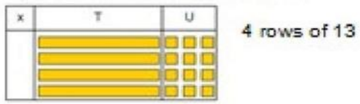
Multiply 2 digit numbers by 1 digit numbers

Concrete

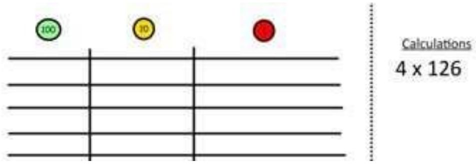
Show the links with arrays to first introduce the grid method.

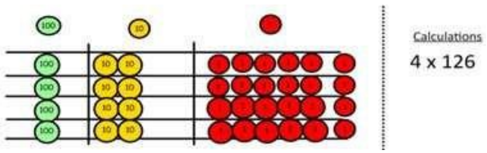


Move onto base ten to move towards a more compact method.



Move on to place value counters to show how we are finding groups of a number. We are multiplying by 4 so we need 4 rows

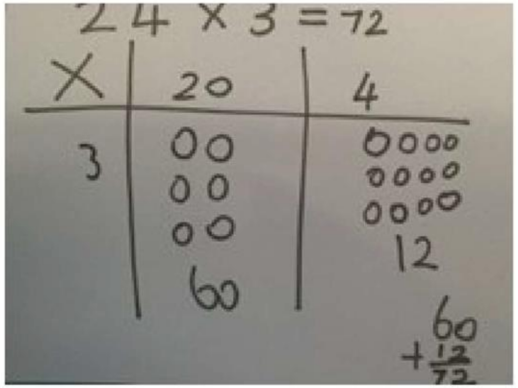




Pictorial

Children can represent their work with place value counters in a way that they understand.

They can draw the counters using colours to show different amounts or just use the circles in the different columns to show their thinking as shown below.



Bar model are used to explore missing numbers

Abstract

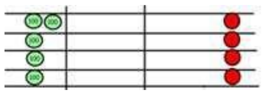
Start with multiplying by one digit numbers and showing the clear addition alongside the grid.

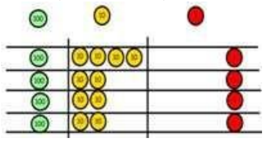
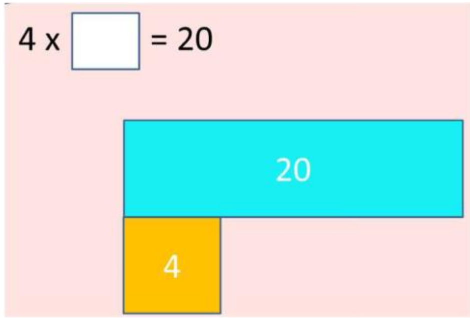
×	30	5
7	210	35

210 + 35 = 245

Move forward to the formal written method:

$$\begin{array}{r} 35 \\ \times\ 7 \\ \hline 245 \\ \hline {\scriptstyle 3} \end{array}$$



	Fill each row with 126. Add up each column, starting with the ones making any exchanges needed Then you have your answer. 		
Solve problems, including missing number problems, integer scaling problems,			Three times as high, eight times as long ? x 5 = 20 20 ÷ ? = 5 3 hats and 4 coats, how many different outfits?

YEARS 4 – 6 Multiplication

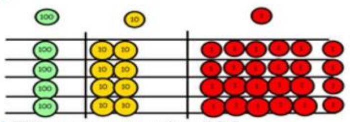
Objective /Strategy	Concrete	Pictorial	Abstract

Grid method recap from year 3 for 2 digits x 1 digit

Move to multiplying 3 digit numbers by 1 digit. (year 4 expectation)

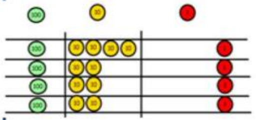
Concrete

Use place value counters to show how we are finding groups of a number. We are multiplying by 4 so we need 4 rows



Calculations
4 x 126

Fill each row with 126

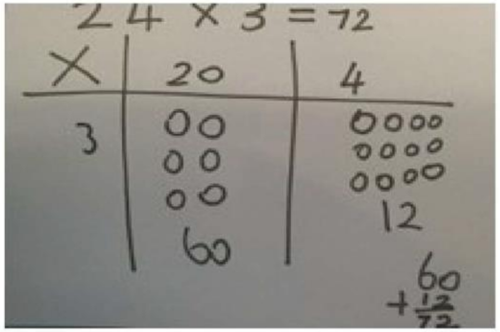


Add up each colu nes making any exchanges needed

Pictorial

Children can represent their work with place value counters in a way that they understand.

They can draw the counters using colours to show different amounts or just use the circles in the different columns to show their thinking as shown below.



Abstract

Start with multiplying by one digit numbers and showing the clear addition alongside the grid.

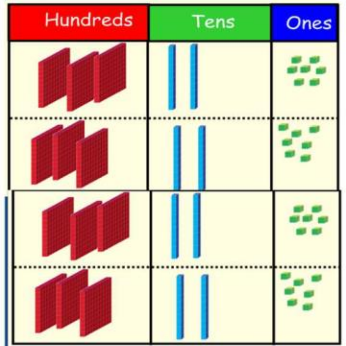
×	30	5
7	210	35

210 + 35 = 245

Column multiplication

Concrete

Children can continue to be supported by place value counters at the stage of multiplication. This initially done where there is no regrouping.   321 x 2 = 642



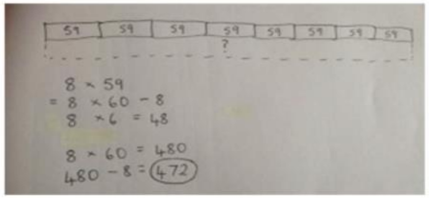
It is important at this stage that they always multiply the ones first.

The corresponding long multiplication is modelled alongside

Pictorial

x	300	20	7
4	1200	80	28

The grid method my be used to show how this relates to a formal written method.



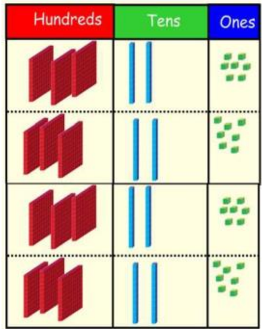
Bar modelling and number lines can support learners when solving problems with multiplication alongside the formal written methods.

Abstract

$$\begin{array}{r} 327 \\ \times \quad 4 \\ \hline 28 \\ 80 \\ 1200 \\ \hline 1308 \end{array}$$

$$\begin{array}{r} 3\,2\,7 \\ \times \qquad 4 \\ \hline 1\,3\,0\,8 \\ {\scriptstyle 1\;\,2\;\,} \end{array}$$

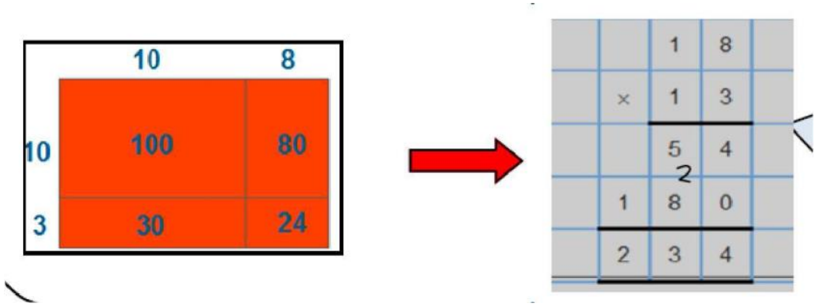
This may lead to a compact method.

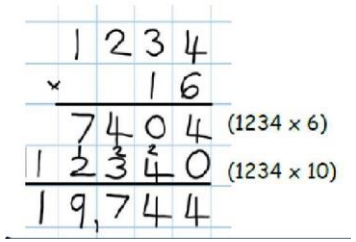
<table>
<tr><th>Objective /Strategy</th><th>Concrete</th><th>Pictorial</th><th>Abstract</th></tr>
<tr>
<td>Column Multiplication for 3 and 4 digits x 1 digit.</td>
<td>

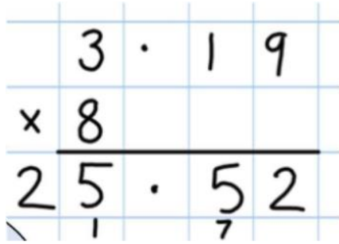
It is important at this stage that they always
Multiply the ones first.
Children can continue to be supported by place value counters at the stage of multiplication. This initially done where there is no regrouping. 321 x 2 = 642</td>
<td><table>
<tr><td>x</td><td>300</td><td>20</td><td>7</td></tr>
<tr><td>4</td><td>1200</td><td>80</td><td>28</td></tr>
</table></td>
<td>327
x 4
______
28
80
120
1308

<table>
<tr><td></td><td>3</td><td>2</td><td>7</td></tr>
<tr><td>x</td><td></td><td></td><td>4</td></tr>
<tr><td>1</td><td>3</td><td>0</td><td>8</td></tr>
<tr><td></td><td>1</td><td>2</td><td></td></tr>
</table></td>
</tr>
</table>

Column multiplication	Manipulatives may still be used with the corresponding long multiplication modelled alongside.	Continue to use bar modelling to support problem solving	18 x 3 on the first row (8 x 3 =24, carrying the 2 for 20, then 1 x 3) 18 x 10 on the 2nd row. Show multiplying by 10 by putting zero in units first



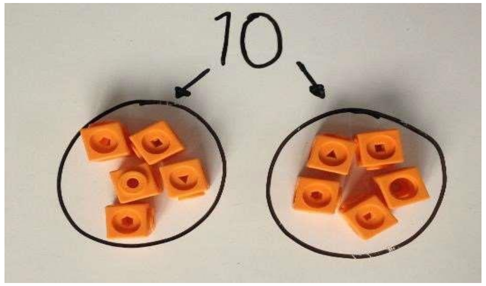
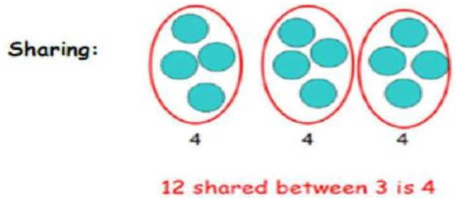


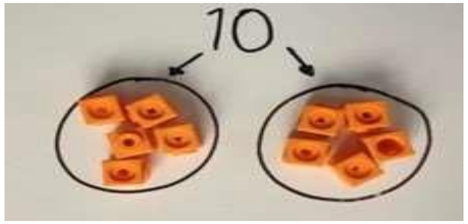
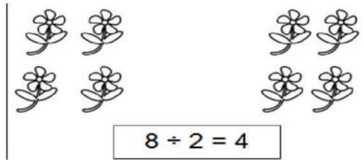
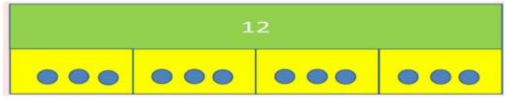
Objective/Strategy	Concrete	Pictorial	Abstract
Multiplying decimals up to 2 decimal places by a single digit.			Remind children that the single digit belongs in the units column. Line up the decimal points in the question and the answer. 

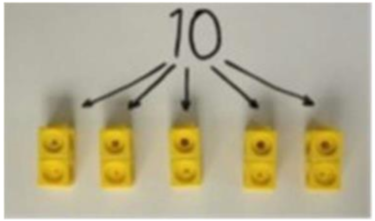
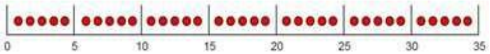
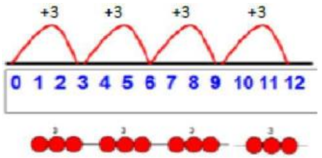
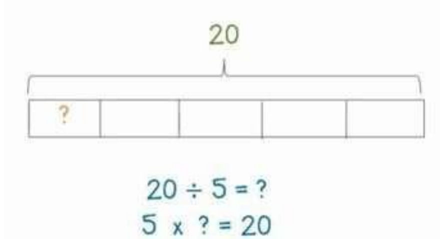
YEAR 1

Objective /Strategy	Concrete	Pictorial	Abstract

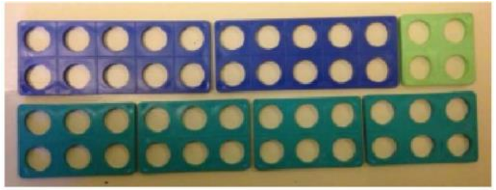
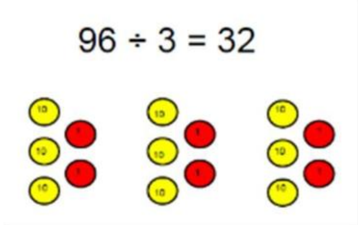
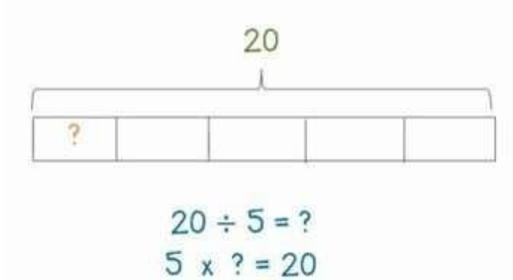
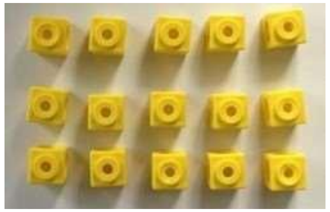
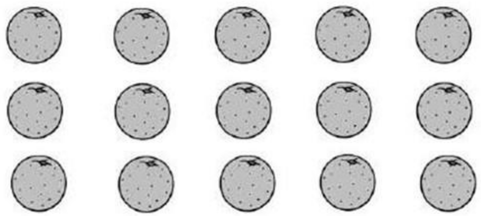
Objective/ Strategy	Concrete	Pictorial	Abstract
	 	 4	

Division as sharing *Use Gordon ITPs for modelling*	 I have 10 cubes, can you share them equally in 2 groups?	Children use pictures or shapes to share quantities. 8 shared between 2 is  	12 shared between 3 is 4

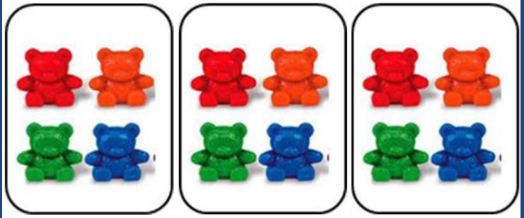
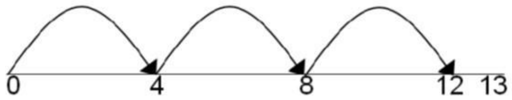
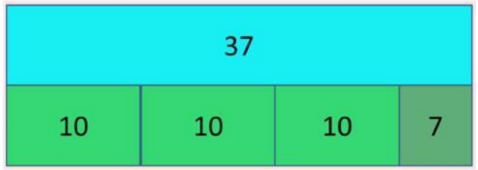
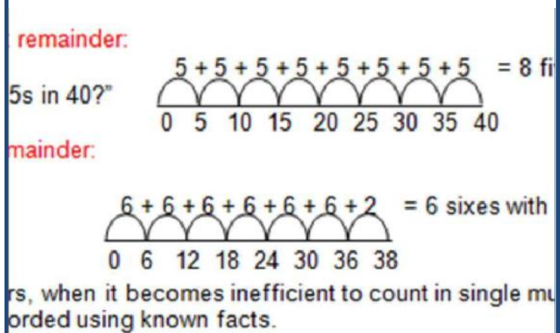
Objective/Strategy	Concrete	Pictorial	Abstract
Division as sharing	 I have 10 cubes, can you share them equally in 2 groups?	Children use pictures or shapes to share quantities.  Children use bar modelling to show and support understanding.  12 ÷ 4 = 3	$12 \div 3 = 4$

Division as grouping	Divide quantities into equal groups. Use cubes, counters, objects or place value counters to aid understanding.  	Use number lines for grouping  12 ÷ 3 = 4 Think of the bar as a whole. Split it into the number of groups you are dividing by and work out how many would be within each group. 	28 ÷ 7 = 4 Divide 28 into 7 groups. How many are in each group?

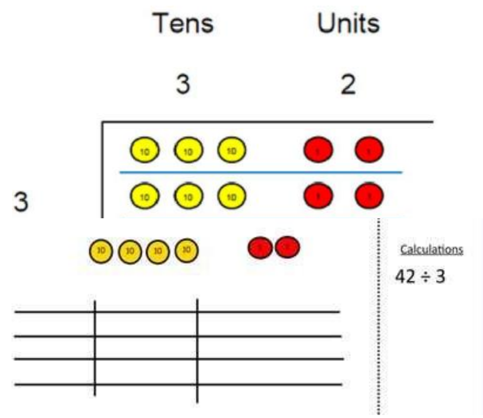
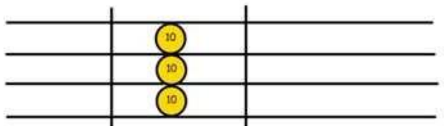
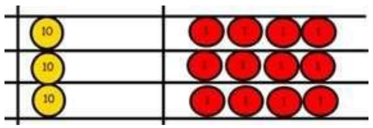
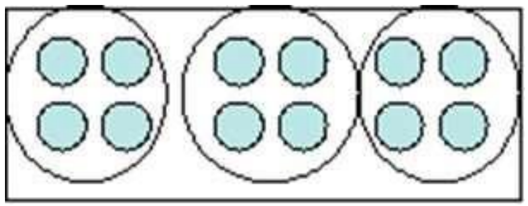
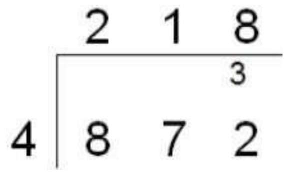
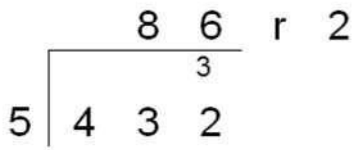
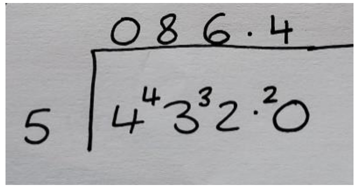
YEAR 2			
Objective/Strategy	**Concrete**	**Pictorial**	**Abstract**

Division as grouping	Use cubes, counters, objects or place value counters to aid understanding.  24 divided into groups of 6 = 4  	Continue to use bar modelling to aid solving division problems.  	How many groups of 6 in 24? 24 ÷ 6 = 4
Division with arrays	 Link division to multiplication by creating an array and thinking about the number sentences that can be created. Eg 15 ÷ 3 = 5 5 x 3 = 15 15 ÷ 5 = 3 3 x 5 = 15	Draw an array and use lines to split the array into groups to make multiplication and division sentences 	Find the inverse of multiplication and division sentences by creating eight linking number sentences. 7 x 4 = 28 4 x 7 = 28 28 ÷ 7 = 4 28 ÷ 4 = 7 28 = 7 x 4 28 = 4 x 7 4 = 28 ÷ 7 7 = 28 ÷ 4

YEAR 3 (Greater Depth Y2)

Objective/Strategy	Concrete	Pictorial	Abstract
Division with remainders.	14 ÷ 3 = Divide objects between groups and see how much is left over  	Jump forward in equal jumps on a number line then see how many more you need to jump to find a remainder.  Draw dots and group them to divide an amount and clearly show a remainder.  Use bar models to show division with remainders.   	Complete written divisions and show the remainder using r. 29 ÷ 8 = 3 REMAINDER 5 ↑ ↑ ↑ ↑ dividend divisor quotient remainder

Year 4-6			
Objective/Strategy	Concrete	Pictorial	Abstract

Divide at least 3 digit numbers by 1 digit. Short Division	96 ÷ 3  Use place value counters to divide using the bus stop method alongside 42 ÷ 3= Start with the biggest place value, we are sharing 40 into three groups. We can put 1 ten in each group and we have 1 ten left over.   We exchange this ten for ten ones and then share the ones equally among the groups.  We look how much in 1 group so the answer is 14.	Students can continue to use drawn diagrams with dots or circles to help them divide numbers into equal groups.  Encourage them to move towards counting in multiples to divide more efficiently.	Begin with divisions that divide equally with no remainder.  Move onto divisions with a remainder.  Finally move into decimal places to divide the total accurately. 

Long Division

Step 1—a remainder in the ones

h t o

0 4 1 R1

4) 1 6 5

4 does not go into 1 (hundred). So combine the 1 hundred with the 6 tens (160).

4 goes into 16 four times.

4 goes into 5 once, leaving a remainder of 1.

th h t o

0 4 0 0 R7

8) 3 2 0 7

8 does not go into 3 of the thousands. So combine the 3 thousands with the 2 hundreds (3,200).

8 goes into 32 four times (3,200 ÷ 8 = 400)
8 goes into 0 zero times (tens).
8 goes into 7 zero times, and leaves a remainder of 7.

Long Division

Step 1 continued...

$$
\begin{array}{r@{\,}r}
 & \text{h t o} \\
 & 0\,6\,1 \\ \cline{2-2}
4 \,) & 2\,4\,7 \\
 & -\,4 \\ \cline{2-2}
 & 3
\end{array}
$$

When dividing the ones, 4 goes into 7 one time. Multiply 1 × 4 = 4, write that four under the 7, and subract. This finds us the remainder of 3.

Check: 4 × 61 + 3 = 247

$$
\begin{array}{r@{\,}r}
 & \text{th h t o} \\
 & 0\,4\,0\,2 \\ \cline{2-2}
4 \,) & 1\,6\,0\,9 \\
 & -\,8 \\ \cline{2-2}
 & 1
\end{array}
$$

When dividing the ones, 4 goes into 9 two times. Multiply 2 × 4 = 8, write that eight under the 9, and subract. This finds us the remainder of 1.

Check: 4 × 402 + 1 = 1,609

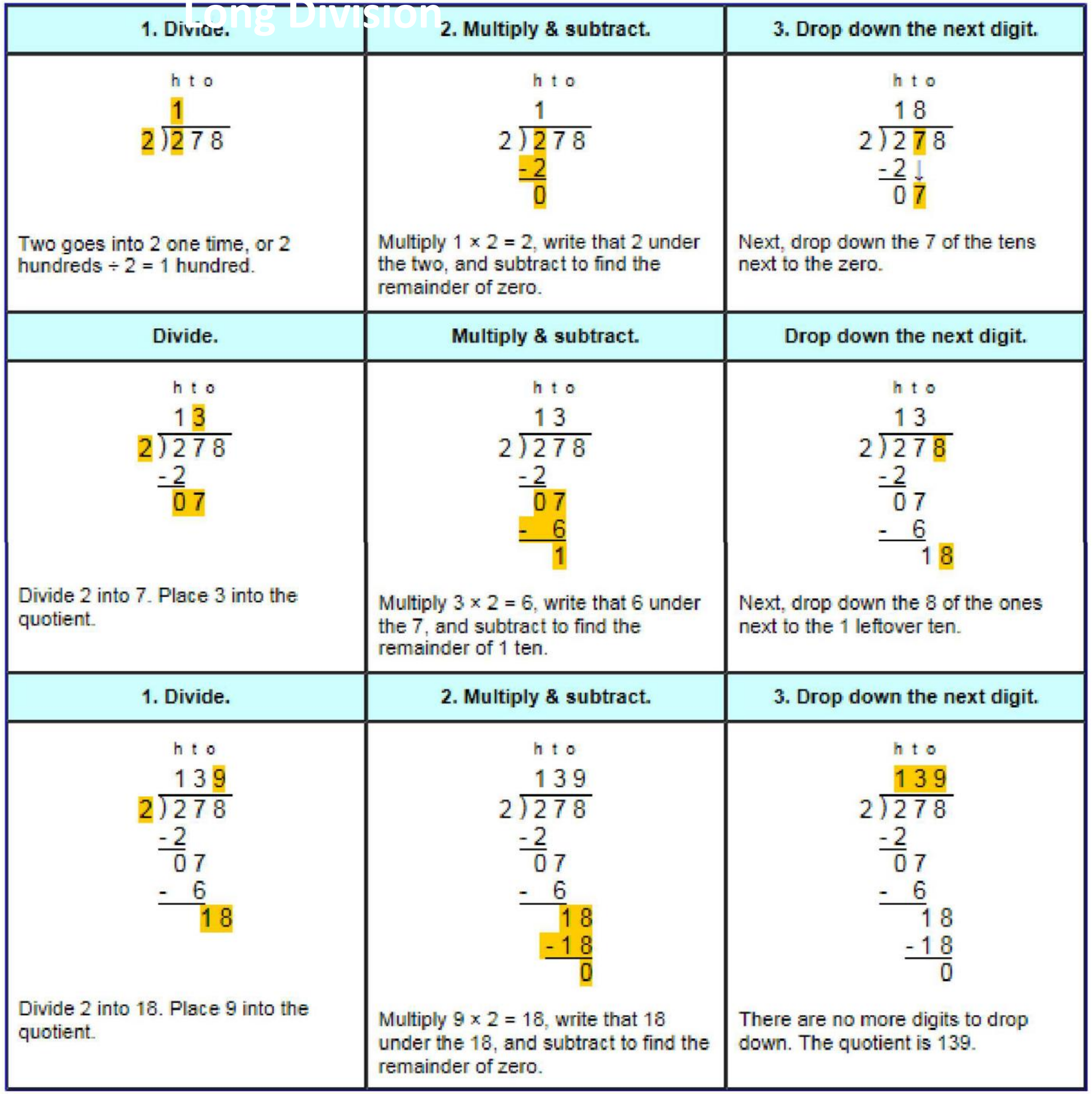
Long Division

Step 2—a remainder in the tens

1. Divide.	2. Multiply & subtract.	3. Drop down the next digit.
t o 2 2) 5 8	t o 2 2) 5 8 - 4 1	t o 2 9 2) 5 8 - 4 ↓ 1 8
Two goes into 5 two times, or 5 tens ÷ 2 = 2 whole tens -- but there is a remainder!	To find it, multiply 2 × 2 = 4, write that 4 under the five, and subtract to find the remainder of 1 ten.	Next, drop down the 8 of the ones next to the leftover 1 ten. You combine the remainder ten with 8 ones, and get 18.

1. Divide.	2. Multiply & subtract.	3. Drop down the next digit.
t o 2 9 2) 5 8 - 4 1 8	t o 2 9 2) 5 8 - 4 1 8 - 1 8 0	t o 2 9 2) 5 8 - 4 1 8 - 1 8 0
Divide 2 into 18. Place 9 into the quotient.	Multiply 9 × 2 = 18, write that 18 under the 18, and subtract.	The division is over since there are no more digits in the dividend. The quotient is 29.

Long Division



Step 2—a remainder in any of the place values